Problem of the noise-noise correlation function in hot non-Abelian plasma

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In this work on the basis of Kadomtsev's kinetic fluctuation theory we present the more general expression for noise-noise correlation function in effective theory for ultrasoft field modes.

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Dynamical processes occurring in systems, described within the framework of Standard Model at finite temperature (probably with the minimal supersymmetric extension) play an essential role in the physics of the early Universe and of heavy ion collisions. In the weak coupling regime hot non-Abelian gauge theories possess several energy scale: the hard scale, corresponding to momentum of order of temperature T; the soft scale $\sim gT$ (g is the gauge coupling) and ultrasoft scale $\sim g^2T$. As long as we are interested in the collective excitations with wavelength $\sim 1/gT$, we can ignore, in leading order in g, collisions among the plasma particles [1]. However the collisions become a dominant effect for color excitations with wavelength $\sim 1/g^2T$.

As known [2] the color fluctuations characterized by the momentum scale g^2T are non-perturbative. Their dynamics is of particular interest, because it is responsible for the large rate of baryon number violation in hot electroweak theory due to topology changing transitions of the weak $SU(2)$ gauge fields [3]. This rate is determined by certain different-time correlation function of the product of two operators, which in turn are a gauge invariant nonlinear functions of the ultrasoft gauge fields $A^a_\mu(X)$. At present the only known instrument to evaluate real time dependent quantities is the classical field approximation [4] and possible extension [5] which contains additional degrees of freedom representing the hard field modes. For time dependent correlation function it was important to find an effective theory for the ultrasoft field modes.

The effective theory at ultrasoft momentum scale ($\omega \sim g^4T$, $|\mathbf{p}| \sim g^2T$) is generated by a Boltzmann-Langevin equation which includes a collision term for color relaxation and the Gaussian noise term, which keeps the ultrasoft modes in thermal equilibrium. The Boltzmann-Langevin equation has been obtained by different approaches. The first is connected with Bödeker's effective theory for $|\mathbf{p}| \ll gT$ field modes [6, 7]. Starting from the collisionless non-Abelian Vlasov equation, that is the result of integrating out the scale T [1], Bödeker has shown how one can integrate out the scale gT in an expansion in the coupling g. At leading order in g, he obtained the linearized Vlasov-Boltzmann equation for the hard field modes, which besides a collision term also includes a Gaussian noise arising from thermal fluctuations of initial conditions of the soft fields. Afterwards, an alternative derivation of the Boltzmann-Langevin equation was proposed by Litim and Manuel [8, 9]. The authors used a classical transport theory in the spirit of Heinz [10]. The approach of Litim and Manuel [8, 9] provides not only the correct collision term but also the correct noise-noise correlator. This correlator was obtained, similar to Bödeker [6], directly from the microscopic theory without making use of the fluctuation-dissipation theorem. A somewhat different approach of more phenomenological character to the computation of the correlator of stochastic source was presented by the same authors in [11], where the well known link between a linearized collision integral and the entropy was exploited.

Blaizot and Iancu [12] presented a detailed derivation of the Vlasov-Boltzmann equation, starting from the Kadanoff-Baym equations. The derivation is based on the method of gauge covariant gradient expansion first proposed by them for the collective dynamics at the scale gT [1]. In work [13] Blaizot and Iancu derived the statistics of the noise term in the Boltzmann-Langevin equation by using the fluctuation-dissipation theorem together with the known structure of the collision term in the Boltzmann equation.

The purpose of this paper is to show that the Boltzmann-Langevin equation in the form obtained by Blaizot and Iancu [12] is somewhat more general, than the equation obtained by Bödeker [6, 7].

We use the metric $g^{\mu\nu} = diag(1, -1, -1, -1)$, choose units such that $c = k_B = 1$ and note $X = (X_0, \mathbf{X})$. On a space-time scale $X \gg (gT)^{-1}$ the ultrasoft colored fluctuation of the gluon density in the adjoint representation $\delta N(\mathbf{k}, X) = \delta N^a(\mathbf{k}, X)T^a$ ($(T^a)^{bc} \equiv -if^{abc}$) satisfies the

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linearized Boltzmann-Langevin equation[1]

$$[v \cdot D_X, \delta N(\mathbf{k}, X)] + g\mathbf{v} \cdot \mathbf{E}(X)\, \frac{dN(\epsilon_{\mathbf{k}})}{d\epsilon_{\mathbf{k}}} \tag{1}$$

$$= \hat{\mathrm{C}}_{\mathbf{k}} \delta N(\mathbf{k}, X) + y(\mathbf{k}, X).$$

Here, $v = (1, \mathbf{v})$, $\mathbf{v} = \mathbf{k}/|\mathbf{k}|$; $D_\mu = \partial_\mu + igA_\mu(X)$; $[\,,\,]$ denotes a commutator; $\mathbf{k}$ is a momentum of hard thermal gluons; $\mathbf{E}(X) = \mathbf{E}^a(X)T^a$ is a chromoelectric field; $N(\epsilon_{\mathbf{k}}) = 1/(\exp(\epsilon_{\mathbf{k}}/T) - 1)$ is a boson occupation factor, where $\epsilon_{\mathbf{k}} \equiv |\mathbf{k}|$. The collision operator $\hat{\mathrm{C}}_{\mathbf{k}}$ acts on function on the right according to [12]

$$\hat{\mathrm{C}}_{\mathbf{k}} f(\mathbf{k}) \equiv g^4 N_c T \int \frac{d\mathbf{k}'}{(2\pi)^3}\, \Phi(\mathbf{v} \cdot \mathbf{v}') \tag{2}$$

$$\times \left\{ \frac{dN(\epsilon_{\mathbf{k}'})}{d\epsilon_{\mathbf{k}'}}\, [T^a, [T^a, f(\mathbf{k})]] - \frac{dN(\epsilon_{\mathbf{k}})}{d\epsilon_{\mathbf{k}}}\, T^a\, \mathrm{Tr}\,(T^a f(\mathbf{k}')) \right\},$$

where the collision kernel $\Phi(\mathbf{v} \cdot \mathbf{v}')$ reads

$$\Phi(\mathbf{v}{\cdot}\mathbf{v}') \simeq \frac{2}{\pi^2 m_D^2} \frac{(\mathbf{v} \cdot \mathbf{v}')^2}{\sqrt{1 - (\mathbf{v} \cdot \mathbf{v}')^2}} \ln\left(\frac{1}{g}\right),\; m_D^2 = \frac{1}{3}\, g^2 N_c T^2$$

within logarithmic accuracy. The function $y(\mathbf{k}, X) = y^a(\mathbf{k}, X)T^a$ on the right-hand side of Eq. (1) is a noise term. This term injects energy compensating the energy loss at scale $g^2 T$ by virtue of the damping term.

Furthermore we write out a general expression for a correlation function of the noise term $y(\mathbf{k}, X)$ in the form proposed by Kadomtsev [14] with a minimal extension to the color degrees of freedom

$$\ll y^a(\mathbf{k}, X)T^a \otimes y^b(\mathbf{k}', X')T^b \gg$$

$$= -\frac{1}{2N_c} \left(\hat{\mathrm{C}}_{\mathbf{k}} \otimes \hat{\mathrm{I}} + \hat{\mathrm{I}} \otimes \hat{\mathrm{C}}_{\mathbf{k}'} \right) T^a \otimes T^a \tag{3}$$

$$\times\, (2\pi)^3 \delta^{(3)}(\mathbf{k} - \mathbf{k}') N(\epsilon_{\mathbf{k}'})[1 + N(\epsilon_{\mathbf{k}'})]\, \delta^{(4)}(X - X').$$

Here, a symbol $\otimes$ denotes a direct production in a color space, $\hat{\mathrm{I}}$ is an identity operator. We note that in original work of Kadomtsev [14] the factor $N(\epsilon_{\mathbf{k}'})$ in noise-noise correlation function stands instead of the factor $N(\epsilon_{\mathbf{k}'})[1 + N(\epsilon_{\mathbf{k}'})] = -T(dN(\epsilon_{\mathbf{k}'})/d\epsilon_{\mathbf{k}'})$. In [14] pure classical gas with Maxwell-Boltzmann statistic was considered, while we consider a hot quantum plasma for gluons (in the semiclassical limit), which obey Bose-Einstein statistic. Using the definition of collision term (2) and decomposing the momentum δ-function in polar coordinates

$$\delta^{(3)}(\mathbf{k} - \mathbf{k}') = \frac{1}{4\pi \mathbf{k}^2}\, \delta(|\mathbf{k}| - |\mathbf{k}'|)\, \delta^{(S^2)}(\mathbf{v} - \mathbf{v}'),$$

where $\delta^{(S^2)}(\mathbf{v} - \mathbf{v}')$ is a delta-function on a unit sphere, from (3) we obtain the following expression for the noise-noise correlation function

[1] This equation is taken in the form suggested by Blaizot and Iancu in Ref. [12].

$$\ll y^a(\mathbf{k}, X) y^b(\mathbf{k}', X') \gg = -\,(2\pi)^3 \delta^{ab} \frac{T}{N_c} \left\{ \gamma\, \frac{dN(\epsilon_{\mathbf{k}})}{d\epsilon_{\mathbf{k}}}\, \frac{1}{4\pi \mathbf{k}^2}\, \delta(|\mathbf{k}|{-}|\mathbf{k}'|)\, \delta^{(S^2)}(\mathbf{v}{-}\mathbf{v}') + \frac{g^4 N_c^2 T}{(2\pi)^3}\, \frac{dN(\epsilon_{\mathbf{k}})}{d\epsilon_{\mathbf{k}}}\, \frac{dN(\epsilon_{\mathbf{k}'})}{d\epsilon_{\mathbf{k}'}}\, \Phi(\mathbf{v}{\cdot}\mathbf{v}') \right\}$$

$$\times\, \delta^{(4)}(X - X'). \tag{4}$$

Here,

$$\gamma = m_D^2\, \frac{g^2 N_c T}{2} \int \frac{d\Omega_{\mathbf{v}'}}{4\pi}\, \Phi(\mathbf{v}{\cdot}\mathbf{v}') \simeq \frac{g^2 N_c T}{2} \left(\ln\!\left(\frac{m_D}{\mu}\right) + O(1) \right)$$

is the damping rate for a hard transverse gluon with velocity $\mathbf{v}$, and μ is the magnetic screening "mass", usually entered by hand for removal of infrared divergence. The equation (4) is the main result of our report.

From Eq. (4) we see, that the noise term $y(\mathbf{k}, X)$ depends on both the velocity $\mathbf{v}$ (unit vector) and the magnitude $|\mathbf{k}|$ of the momentum in a nontrivial way, and thus generates (by virtue of the Boltzmann-Langevin equation (1)) similar dependence for the fluctuation $\delta N(\mathbf{k}, X)$. Note that Litim and Manuel in [11] (Eqs. (23) and (32)) pointed to the possible nontrivial dependence of noise-noise correlator on the magnitude $|\mathbf{k}|$ of the momentum. However our expression (4) is more complicated, since here contrary to [11], we have different factors (depending on $|\mathbf{k}|$ and $|\mathbf{k}'|$) before functions $\delta^{(S^2)}(\mathbf{v} - \mathbf{v}')$ and $\Phi(\mathbf{v} \cdot \mathbf{v}')$.

For the calculation of the color current

$$j_\mu(X) = 2gN_c \int \frac{d\mathbf{k}}{(2\pi)^3}\, v_\mu \delta N(\mathbf{k},X)$$

we need only the second momentum with respect to the magnitude $|\mathbf{k}|$ of ultrasoft fluctuation $\delta N(\mathbf{k},X)$. For this and similar physical problems, where the second moment is the only relevant quantity, it is convenient to introduce new functions $W(X,\mathbf{v})$ and $\nu(X,\mathbf{v})$ depending on velocity $\mathbf{v}$ only, instead of initial functions $\delta N(\mathbf{k},X)$ and $y(\mathbf{k},X)$, by the rules

$$\int\limits_0^\infty \mathbf{k}^2 d|\mathbf{k}|\, \delta N(\mathbf{k},X) = -g\, W(X,\mathbf{v}) \int\limits_0^\infty \mathbf{k}^2 d|\mathbf{k}|\, \frac{dN(\epsilon_{\mathbf{k}})}{d\epsilon_{\mathbf{k}}}, \tag{5}$$

$$\int\limits_0^\infty \mathbf{k}^2 d|\mathbf{k}|\, y(\mathbf{k},X) = -g\, \nu(X,\mathbf{v}) \int\limits_0^\infty \mathbf{k}^2 d|\mathbf{k}|\, \frac{dN(\epsilon_{\mathbf{k}})}{d\epsilon_{\mathbf{k}}}. \tag{6}$$

Here, the first relation is an extension of usually used parametrization of off-equilibrium fluctuation [12, 15]

$$\delta N(\mathbf{k},X) = -g\, W(X,\mathbf{v})\, \frac{dN(\epsilon_{\mathbf{k}})}{d\epsilon_{\mathbf{k}}},$$

which is valid in the absence of the noise term $y(\mathbf{k},X)$ or when this term depends on velocity $\mathbf{v}$ only. The general connection, Eq. (5), between the functions $\delta N(\mathbf{k},X)$ and $W(X,\mathbf{v})$ involves an integral over $\mathbf{k}$, which reflects the corresponding integral in the relation (6) between the noise term $y(\mathbf{k},X)$ and $\nu(X,\mathbf{v})$.

Multiplying Eq. (1) and correlation function (4) by $\mathbf{k}^2$ and $\mathbf{k}^2\mathbf{k}'^2$, and integrating over $d|\mathbf{k}|$ and $d|\mathbf{k}|d|\mathbf{k}'|$, correspondingly (with regard to (5) and (6)) instead of Eq. (1) – (3) we recover the equations for the functions $W(X,\mathbf{v})$ and $\nu(X,\mathbf{v})$, first proposed by Bödeker [6, 7]. Let us stress that such a reduction of initial system of Eqs. (1) – (3) to simpler system for functions $W(X,\mathbf{v})$ and $\nu(X,\mathbf{v})$ does not lead to any loss of the information, if we only restrict our consideration to the second moment with respect to $|\mathbf{k}|$ of ultrasoft gluon fluctuations $\delta N(\mathbf{k},X)$. But for the calculation of more general quantities, which involve also the other moments of the ultrasoft fluctuations (e.g. correlation function of energy density fluctuation), there is no such one-to-one correspondence between these systems by virtue of nontrivial dependence of noise-noise correlator (4) on magnitudes $|\mathbf{k}|$ and $|\mathbf{k}'|$. In this case it is necessary to use more exact system of equations (1), (2) and (4).

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